Online Supplement S-4

Example of a MASEM using 2 empirical correlation matrices

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# Step 1 ########## primary study 1 (Ashforth & Saks, 1996) & primary study 2 (Bechtold et al., 1981) ####################
N01 <- 222                                  # Ashforth & Saks (1996): Sample size was 222.
corr01 <- matrix(c(1.00, 0.11, 0.64, 0.14,  # The correlations reported by Ashforth & Saks.
                   0.11, 1.00, 0.20, 0.69,
                   0.64, 0.20, 1.00, 0.21
                   0.14, 0.69, 0.21, 1.00),
                   4,4)                     # It is a 4 x 4 matrix (2 manifest variables at 2 waves).
N02 <- 64                                   # Bechtold et al. (1981): Sample size was 64.
corr02 <- matrix(c(1.00, 0.15, 0.53, 0.17   # The correlations reported by Bechtold et al.
                   0.15, 1.00, 0.19, 0.57,
                   0.53, 0.19, 1.00, 0.21,
                   0.17, 0.57, 0.21, 1.00),
                   4, 4)                    # It is a 4 x 4 matrix (2 manifest variables at 2 waves).

sampleSizes <- c(N01, N02)                  # Combine information about sample sizes.
noOfStudies <- 2                            # Store the number of studies in R object.

# Step 2 ######### Random Effects Computations ##############################################################
randomCorrAll <- diag(1, 4, 4)                     # Matrix to store the random effect correlations (1 already in diagonal).

for (i in 2:4) {                                   # Loop through the rows of the correlation matrix.
  for (h in 1:(i-1)) {                             # Loop through the columns of the correlation matrix (lower triangle only).
    correlations <- c(corr01[h,i], corr02[h,i])    # Vector of respective [row, column] correlations of all primary studies.

    SEs <- (1-correlations^2)/((sampleSizes-1)^.5) # Standard error of primary study correlations.
    precision <- 1/(SEs^2)                         # Precision of primary study correlations.

    Q <- sum(precision * correlations^2)-          # Compute Q.
         (sum(precision * correlations))^2/sum(precision)
    c <- sum(precision)-sum(precision^2)/sum(precision) # Intermediate computational step.

    tau2 <- (Q-(noOfStudies-1))/c                  # Compute tau square.
    if (Q < (noOfStudies-1)) tau2 <- 0             # If Q < df set tau square to 0.

    tau2Extended <- cbind(replicate(noOfStudies, tau2)) # Intermediate computational step.
    Ttot_Weights <-colSums(1/ (SEs^2 + tau2Extended)) # Intermediate computational step.
    Ttot_Means <- colSums(correlations * 1/        # Intermediate computational step.
         (SEs^2 + tau2Extended))
    randomCorrAll[h,i] <- Ttot_Means/Ttot_Weights  # Compute random effect and insert in upper triangular matrix.
    randomCorrAll[i,h] <- Ttot_Means/Ttot_Weights  # Compute random effect and insert in lower triangular matrix.
  }      # Close i-loop (rows).
}        # Close h-loop (columns).

# Step 3 ######### MASEM (with lavaan) ########################################################################
require("lavaan")                                  # Load lavaan to perform SEM

model1 <- '                                        # Specify SEM.
     S1 =~ JS1                                     # Let the indicator ‘JS1’ load on latent factor S1.
     S2 =~ JS2                                     # Let the indicator ‘JS2’ load on latent factor S2.
     P1 =~ IP1                                     # Let the indicator ‘IP1’ load on latent factor P1.
     P2 =~ IP2                                     # Let the indicator ‘IP2’ load on latent factor P2.
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43.         S2 ~ S1 + P1                                  # Regress S2 on S1 and P1.
44.         P2 ~ P1 + S1                                  # Regress P2 on P1 and S1.
45.         P1 ~~ S1                                      # Let P1 and S1 correlate freely.
46.         P2 ~~ S2                                      # Let the structural disturbance of P2 and S2 correlate freely.
47. '         # End model specification.

48. cor.lavaan <- getCov(paste(randomCorrAll[upper.tri(     # Translate aggregated random effect matrix so that lavaan can use it.
49.         randomCorrAll, diag=T)],
50.                    sep='', collapse=" "),
51.               names = c("JS1", "IP1", "JS2", "IP2"))
52. model1.fit1 <- sem(model1, sample.cov = cor.lavaan,     # Fit SEM.
53.                    sample.nobs = sum(sampleSizes))
54. summary(model1.fit1)                                    # Display model fit.
```

A MASEM could be performed in three steps, which correspond to the three script sections above. In Section 1 (#1-15), information about primary study data are imported containing information on sample size (N) and the and the correlation matrix (corr).
In Section 2 (#16-35), these primary study data are then used to compute the aggregated random effects correlations. For this purpose, we repeat (loop) all computational steps for the elements of the lower triangular matrix. The equations (cf., Borenstein, Hedges & Rothstein, 2007; Borenstein et al., 2009) involve estimating Q first. To assign initial weight to the studies, the standard errors of their correlations are used, which is

$$s_i = \frac{\left(1 - r_i^2\right)}{\sqrt{N_i - 1}} \tag{10}$$

The weight assigned to each study (called precision in the script) is the inverse of the squared standard error:

$$w_i = \frac{1}{s_i^2} \tag{11}$$

Then Q is computed as follows (Borenstein et al., 2009):

$$Q = \sum_{i=1}^{k} w_i r_i^2 - \frac{\left(\sum_{i=1}^{k} w_i r_i\right)^2}{\sum_{i=1}^{k} w_i} \tag{12}$$

When defining C as

$$C = \sum_{i=1}^{k} w_i - \frac{\sum_{i=1}^{k} w_i^2}{\sum_{i=1}^{k} w_i} \tag{13}$$

then the between study variance τ^2 can be estimated using the method of moments:

$$\tau^2 = \frac{Q - df}{C} \text{ if } Q > df \tag{14}$$

or

$$\tau^2 = 0 \text{ if } Q < df\,. \tag{15}$$

Finally, in order to aggregate the random effects, the correlations of the primary studies are estimated by the inverse of the sum of their respective standard errors plus τ^2.

$$\rho = \frac{\sum_{i=1}^{k} r_i \frac{1}{s_i^2 + \tau^2}}{\sum_{i=1}^{k} \frac{1}{s_i^2 + \tau^2}} \tag{16}$$

In Section 3 (#36-54), the lavaan package is installed. The previously computed random effects correlation matrix is translated into a format that can be read by lavaan. Then to model is fitted to the aggregated random effects correlation matrix. Finally, the estimated parameters are displayed.